OPTIMIZATION THEORY AND ADVANCED NUMERICAL METHODS

Faculty of Electronics
Embedded Robotics
M.Sc. level

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Outlines of lectures

- Introduction
- Optimization problem definition and classification
- Linear programming problem – the solution methods for continuous variables
- Linear programming methods for integer variables
- Nonlinear programming methods :
  - Optimization methods without constraints
  - Optimization methods with constraints
- Local optimization methods – , Nelder-Meade algorithm, non-gradient algorithms, gradient algorithms
- Global optimization methods– (Genetic algorithms – GA, Evolution algorithms – EA, Ant Colony algorithms , Particle Swarm optimization algorithms, Harmony Search algorithms – HSA).
- Multi-criterial optimization problems

Literature

- Nocedal J. and Wright S., Numerical optimization, 2006.
- Ruszczyński ., Nonlinear optimization, Princeton University Press, 2006.
- Fletcher R., Practical methods of optimization, J. Wiley Ed. 2000.
- Minoux M., Mathematical programming – Theory and algorithms, J. Wiley &Sons Ed. 2008.
- Rardin R.L., Optimization in operations research. Prentice Hall Ed. 1998.
- Stephan Boyd, Convex optimization, Cambridge University Press, 2004 (bv_cvxbook.pdf)
- Thomas Weise, Global optimization algorithms – Theory and Applications (book.pdf)
- Eckart Zitzler: *Evolutionary Algorithms for Multiobjective Optimization: Methods and Applications,* Zürich 1999.
- Jeffrey Horn, Nicholas Nafpliotis, David E. Goldberg: *A Niche Pareto Genetic Algorithm for Multiobjective Optimalization,* IEEE 1994 , Volume 1, pp. 82-87.
- http://delta.cs.cinvestav.mx/~ccoello/EMOO/

Optimization of the multi-dimensional programming problem

Desicion variables vector **x**: $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$

where: n – number of decision variables.

Objective function f(x) : $f(\mathbf{x}) : R^n \longrightarrow R^1$

and m constraint functions $g_i(x)$:

$$g_i(\mathbf{x}) : R^n \longrightarrow R^1 \quad dla \quad i = 1, \ldots, m$$

Let us find the decision variables vector $\hat{x}$, which belongs to a set X of admissible solutions in the form:

$$X = \{x \mid g_i(\mathbf{x}) \le 0, \quad i = 1, \ldots, m\}$$

such, that for $\forall \mathbf{x} \in X$

$$f(\hat{\mathbf{x}}) \le f(\mathbf{x})$$

It's equivalent to: $\min_{\mathbf{x} \in X} f(\mathbf{x}) = f(\hat{\mathbf{x}})$

Applications

- **Optimization of technological processes : model formulation,**
- **Identification of technological processes**
- **Optimal management of an enterprise – cost minimization**
- **The maximization of consumption, profit maximization or the minimization of waste in a production process**
- **Control of technological processes**

Applications cont.:

- **Optimal flows in networks (water distribution network, gas distribution network, computer network).**
- **Products distribution, optimal scheduling in technological processes.**
- **Cutting stocks problems – algorithms for a furniture industry**
- **Vehicle Routing Problem VRP - VRPTW, CVRP, CVRPTW**

Classification of optimization problems

- Model of the process:
  - Static problem
  - Dynamic problem
- The solution space of decision variables:
  - Real variables
  - Discrete variables (integer variables, binary variables)
  - Mixed variables
- Objective function:
  - Scalar objective function
  - Vector objective function (multi-criteria problem)
- Data necessary for optimization algorithms:
  - Objective function values
  - Gradient of an objective function
  - Hessian of an objective function

Linear programming problem LP

$$\max\ f(\mathbf{x}) = \mathbf{c}^T\mathbf{x}$$

On the set of constraints:

$$\mathbf{A}_1\mathbf{x} \le \mathbf{b}_1$$
$$\mathbf{A}_2\mathbf{x} \ge \mathbf{b}_2$$
$$\mathbf{x} \ge 0$$

dim **x**=n, dim **c**=n

Matrices $\mathbf{A_1}$, $\mathbf{A_2}$ for m_1 and m_2 constraints

dim $\mathbf{A_1}$ =[m $_1$ x n], dim $\mathbf{A_2}$ =[m $_2$ x n]

Vectors b_1, b_2 : dim $\mathbf{b_1}$=m_1, dim $\mathbf{b_2}$=m_2

Quadratic programming problem

$$\max_{\mathbf{x}\in X}\ f(\mathbf{x}) = 0.5\mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x}$$

where:

$$X = \{\mathbf{x} : \mathbf{D}^T\mathbf{x} \le \mathbf{e},\ \ \mathbf{x} \ge 0\}$$

Nonlinear multi-dimentional programming problem with constraints

$$\min\ f(\mathbf{x}) = (x_1 - 2)^2 + (x_2 - 1)^2$$

With two constraints:

$$x_1^2 \le x_2$$
$$x_1 + x_2 \le 2$$

Nonlinear multi-dimentional optimization problem with the set of constraints

$$\min_{x\in X} f(x) = 2(x_1)^2 + 4x_1(x_2)^3 - 10x_2x_1 + (x_2)^2$$

$$X = \{(x_1, x_2) \mid x_1(x_2)^2(2.4 + x_2) \le 3$$
$$\wedge\ 3/2x_1 + x_2 \ge 0$$
$$\wedge\ (-3 \le x_i \le 3, i = 1,2)\}.$$